Lecturered By Dr. H. K. Yadav.
Deptt. of Mathematics Marwari College, TMBU.
B.Sc.-P-I(H), Paper-I, Date-01-05-2020.
Algebra.

Q. 1. Define subgroup and prove that if H be a subgroup of a group G, prove that the identity of H is the same as that of G.

Soln: Subgroup:→ ~~Let (G,o)~~ Let (G, o) be any group and H any sub-set of G. If the set H together with the operation o forms a group (H, o), then (H, o) is a sub-group of (G, o). For example.

The additive group of even integers H is a sub-group of the additive group of integers G.

Proof: Let e and e' be the identities of G and H respectively. Then we have to prove that e' = e.

Let $a \in H$. Then $e' o = a$ [by identity axiom]

Since, H is a subgroup of the group G, therefore

$a \in H \Rightarrow a \in G$

$\Rightarrow ea = a$ [by identity axiom]

Hence in G, we have

$e'a = ea \Rightarrow e' = e$ [by right cancellation law]

Proved.

Q. 2. prove that if H_1 and H_2 are two subgroups of G under the operation 'o' then $H_1 \cap H_2$ is also a subgroup of G.

Proof: Since H_1 and H_2 are two subgroups of the group G under the operation 'o'. We have to prove that $H_1 \cap H_2$ is also a subgroup of G.

Let $a, b \in H_1 \cap H_2$
$\Rightarrow a, b \in H_1$ and $a, b \in H_2$.

As $b \in H_1$ and H_1 is a group itself, $b^{-1} \in H_1$

So, $a, b \in H_1 \Rightarrow a, b^{-1} \in H_1$
$\Rightarrow a o b^{-1} \in H_1$. Similarly $a o b^{-1} \in H_2$.

$\therefore a, b \in H_1 \cap H_2 \Rightarrow a o b^{-1} \in H_1 \cap H_2$

$\therefore H_1 \cap H_2$ is a subgroup of G.

Proved.

Scanned with CamScanner

Q.3. Let G be a group and N is a normal subgroup of G. Let f be a mapping from G to G/N defined by

$f(x) = Nx$, $\forall x \in G$.

Then f is a homomorphism of G onto G/N and Kernel $f = N$.

Proof: When we define a mapping

$f : G \to G/N$ such that $f(x) = Nx$, $\forall x \in G$.

Let $Nx \in G/N$, then $x \in G$.

We have, $f(x) = Nx \Rightarrow f$ is onto G/N

Let $a, b \in G$, then $f(ab) = Nab = (Na)(Nb) = f(a) \cdot f(b)$

[$\because N$ is normal]

$\therefore f$ is a homomorphism of G onto G/N.

Thus, every quotient group of a Group is a homomorphic image of the group.

Let K be the Kernel of f. The identity of the quotient group G/N is the coset N.

So, $K = \{y \in G : f(y) = N\}$

We have to prove that $K = N$

Let $K \in K$, then $f(K) = N$ i.e. identity of G/N

By definition of f. $f(K) = NK$.

Now, $NK = N \Rightarrow K \in N \Rightarrow K \subseteq N$. —— (I)

Now, let $n \in N$. Then $Nn = N$

We have, $f(n) = Nn = N \Rightarrow n \in K$

$\therefore N \subseteq K$ —— II

From I and II, we conclude that

$N = K$. <u>proved</u>

Q.4. Define Coset.

Soln: Let G be a Group, H a sub-group of G and let $a \in G$. We define two sub-sets of G, to be denoted by aH and Ha, as follows:

$aH = \{ah_1, ah_2, \ldots\}$ and $Ha = \{h_1a, h_2a, \ldots\}$, where $h_1, h_2, \ldots \in H$.

The sub-set aH so obtained is called a left coset of H in G and the sub-set Ha is called a right coset of H in G.

In general $aH \neq Ha$